Algebra 1 Skills Needed to be Successful in Algebra 2

A. Simplifying Polynomial Expressions
   Objectives: The student will be able to:
   • Apply the appropriate arithmetic operations and algebraic properties needed to simplify an algebraic expression.
   • Simplify polynomial expressions using addition and subtraction.
   • Multiply a monomial and polynomial.

B. Solving Equations
   Objectives: The student will be able to:
   • Solve multi-step equations.
   • Solve a literal equation for a specific variable, and use formulas to solve problems.

C. Rules of Exponents
   Objectives: The student will be able to:
   • Simplify expressions using the laws of exponents.
   • Evaluate powers that have zero or negative exponents.

D. Binomial Multiplication
   Objectives: The student will be able to:
   • Multiply two binomials.

E. Factoring
   Objectives: The student will be able to:
   • Identify the greatest common factor of the terms of a polynomial expression.
   • Express a polynomial as a product of a monomial and a polynomial.
   • Find all factors of the quadratic expression $ax^2 + bx + c$ by factoring and graphing.

F. Radicals
   Objectives: The student will be able to:
   • Simplify radical expressions.

G. Graphing Lines
   Objectives: The student will be able to:
   • Identify and calculate the slope of a line.
   • Graph linear equations using a variety of methods.
   • Determine the equation of a line.

H. Regression and Use of the Graphing Calculator
   Objectives: The student will be able to:
   • Draw a scatter plot, find the line of best fit, and use it to make predictions.
   • Graph and interpret real-world situations using linear models.
A. Simplifying Polynomial Expressions

I. Combining Like Terms

- You can add or subtract terms that are considered "like", or terms that have the same variable(s) with the same exponent(s).

Ex. 1: \(5x - 7y + 10x + 3y\)
\[5x - 7y + 10x + 3y\]
\[15x - 4y\]

Ex. 2: \(-8h^2 + 10h^3 - 12h^2 - 15h^3\)
\[-8h^2 + 10h^3 - 12h^2 - 15h^3\]
\[-20h^2 - 5h^3\]

II. Applying the Distributive Property

- Every term inside the parentheses is multiplied by the term outside of the parentheses.

Ex. 1: \(3(9x - 4)\)
\[3 \cdot 9x - 3 \cdot 4\]
\[27x - 12\]

Ex. 2: \(4x^2(5x^3 + 6x)\)
\[4x^2 \cdot 5x^3 + 4x^2 \cdot 6x\]
\[20x^5 + 24x^3\]

III. Combining Like Terms AND the Distributive Property (Problems with a Mix!)

- Sometimes problems will require you to distribute AND combine like terms!!

Ex. 1: \(3(4x - 2) + 13x\)
\[3 \cdot 4x - 3 \cdot 2 + 13x\]
\[12x - 6 + 13x\]
\[25x - 6\]

Ex. 2: \(3(12x - 5) - 9(-7 + 10x)\)
\[3 \cdot 12x - 3 \cdot 5 - 9(-7) - 9(10x)\]
\[36x - 15 + 63 - 90x\]
\[-54x + 48\]
PRACTICE SET 1

Simplify.

1. $8x - 9y + 16x + 12y$

2. $14y + 22 - 15y^2 + 23y$

3. $5n - (3 - 4n)$

4. $- 2(11b - 3)$

5. $10q(16x + 11)$

6. $-(5x - 6)$

7. $3(18z - 4w) + 2(10z - 6w)$

8. $(8c + 3) + 12(4c - 10)$

9. $9(6x - 2) - 3(9x^2 - 3)$

10. $-(y - x) + 6(5x + 7)$
B. Solving Equations

I. Solving Two-Step Equations

A couple of hints:

1. To solve an equation, UNDO the order of operations and work in the reverse order.
2. REMEMBER! Addition is “undone” by subtraction, and vice versa. Multiplication is “undone” by division, and vice versa.

Ex. 1: \(4x - 2 = 30\)

\[
\begin{align*}
+ 2 & \quad + 2 \\
4x & = 32 \\
\div 4 & \quad \div 4
\end{align*}
\]

\[x = 8\]

Ex. 2: \(87 = -11x + 21\)

\[
\begin{align*}
- 21 & \quad - 21 \\
66 & = -11x \\
\div -11 & \quad \div -11
\end{align*}
\]

\[-6 = x\]

II. Solving Multi-step Equations With Variables on Both Sides of the Equal Sign

- When solving equations with variables on both sides of the equal sign, be sure to get all terms with variables on one side and all the terms without variables on the other side.

Ex. 3: \(8x + 4 = 4x + 28\)

\[
\begin{align*}
- 4 & \quad - 4 \\
8x & = 4x + 24 \\
- 4x & \quad - 4x \\
4x & = 24 \\
\div 4 & \quad \div 4
\end{align*}
\]

\[x = 6\]

III. Solving Equations that need to be simplified first

- In some equations, you will need to combine like terms and/or use the distributive property to simplify each side of the equation, and then begin to solve it.

Ex. 4: \(5(4x - 7) = 8x + 45 + 2x\)

\[
\begin{align*}
20x - 35 & = 10x + 45 \\
-10x & \quad -10x \\
10x - 35 & = 45 \\
+ 35 & \quad + 35 \\
10x & = 80 \\
+ 10 & \quad + 10 \\
x & = 8
\end{align*}
\]
PRACTICE SET 2

Solve each equation. You must show all work.

1. \(5x - 2 = 33\)  
   \[2. \quad 140 = 4x + 36\]

3. \(8(3x - 4) = 196\)  
   \[4. \quad 45x - 720 + 15x = 60\]

5. \(132 = 4(12x - 9)\)  
   \[6. \quad 198 = 154 + 7x - 68\]

7. \(-131 = -5(3x - 8) + 6x\)  
   \[8. \quad -7x - 10 = 18 + 3x\]

9. \(12x + 8 - 15 = -2(3x - 82)\)  
   \[10. \quad -(12x - 6) = 12x + 6\]

IV. Solving Literal Equations

- A literal equation is an equation that contains more than one variable.
- You can solve a literal equation for one of the variables by getting that variable by itself (isolating the specified variable).

\[Ex. 1: \quad 3xy = 18, \quad \text{Solve for } x.\]

\[\frac{3xy}{3y} = \frac{18}{3y}\]
\[x = \frac{6}{y}\]

\[Ex. 2: \quad 5a - 10b = 20, \quad \text{Solve for } a.\]

\[+10b = +10b\]
\[5a = 20 + 10b\]
\[\frac{5a}{5} = \frac{20 + 10b}{5}\]
\[a = 4 + 2b\]
PRACTICE SET 3

Solve each equation for the specified variable.

1. \( Y + V = W \), for \( V \)
2. \( 9wr = 81 \), for \( w \)

3. \( 2d - 3f = 9 \), for \( f \)
4. \( dx + t = 10 \), for \( x \)

5. \( P = (g - 9)180 \), for \( g \)
6. \( 4x + y - 5h = 10y + u \), for \( x \)
C. Rules of Exponents

Multiplication: Recall \((x^m)(x^n) = x^{m+n}\)
\[Ex: (3x^4y^2)(4xy^5) = (3 \cdot 4)(x^4 \cdot x^1)(y^2 \cdot y^5) = 12x^5y^7\]

Division: Recall \(\frac{x^m}{x^n} = x^{m-n}\)
\[Ex: \frac{42m^5j^2}{-3m^3j} = \left(\frac{42}{-3}\right)\left(\frac{m^5}{m^3}\right)\left(\frac{j^2}{j^1}\right) = -14m^2j\]

Powers: Recall \((x^m)^n = x^{mn}\)
\[Ex: (-2a^3bc^4)^3 = (-2)^3(a^3)^3(b^1)^3(c^4)^3 = -8a^9b^3c^{12}\]

Power of Zero: Recall \(x^0 = 1, x \neq 0\)
\[Ex: 5x^0y^4 = (5)(1)(y^4) = 5y^4\]

**PRACTICE SET 4**

Simplify each expression.

1. \((c^5)(c)(c^2)\)
2. \(\frac{m^{15}}{m^3}\)
3. \((k^4)^5\)

4. \(d^6\)
5. \((p^4q^2)(p^7q^5)\)
6. \(\frac{45y^3z^{10}}{5y^9z}\)

7. \((-t^7)^3\)
8. \(3f^3g^0\)
9. \((4h^5k^3)(15k^2h^3)\)

10. \(\frac{12a^4b^6}{36ab^2c}\)
11. \((3m^2n)^4\)
12. \((12x^2y)^0\)

13. \((-5a^2b)(2ab^2c)(-3b)\)
14. \(4x(2x^2y)^0\)
15. \((3x^4y)(2y^2)^3\)
D. Binomial Multiplication

I. Reviewing the Distributive Property

The distributive property is used when you want to multiply a single term by an expression.

\[ \text{Ex 1: } 8(5x^2 - 9x) \]
\[ 8 \cdot 5x^2 + 8 \cdot (-9x) \]
\[ 40x^2 - 72x \]

II. Multiplying Binomials – the FOIL method

When multiplying two binomials (an expression with two terms), we use the “FOIL” method. The “FOIL” method uses the distributive property twice!

FOIL is the order in which you will multiply your terms.

First
Outer
Inner
Last

\[ \text{Ex. 1: } (x + 6)(x + 10) \]

\[ (x + 6)(x + 10) \]

\[ \text{First} \quad x \cdot x \quad \longrightarrow \quad x^2 \]
\[ \text{Outer} \quad x \cdot 10 \quad \longrightarrow \quad 10x \]
\[ \text{Inner} \quad 6 \cdot x \quad \longrightarrow \quad 6x \]
\[ \text{Last} \quad 6 \cdot 10 \quad \longrightarrow \quad 60 \]

\[ x^2 + 10x + 6x + 60 \]
\[ x^2 + 16x + 60 \]

(After combining like terms)
Recall: \(4^2 = 4 \cdot 4\)
\[x^2 = x \cdot x\]

Ex. \((x + 5)^2\)
\[(x + 5)^2 = (x + 5)(x+5)\]  Now you can use the “FOIL” method to get a simplified expression.

**PRACTICE SET 5**

Multiply. Write your answer in simplest form.

1. \((x + 10)(x - 9)\)

2. \((x + 7)(x - 12)\)

3. \((x - 10)(x - 2)\)

4. \((x - 8)(x + 81)\)

5. \((2x - 1)(4x + 3)\)

6. \((-2x + 10)(-9x + 5)\)

7. \((-3x - 4)(2x + 4)\)

8. \((x + 10)^2\)

9. \((-x + 5)^2\)

10. \((2x - 3)^2\)
E. Factoring

I. Using the Greatest Common Factor (GCF) to Factor.

- Always determine whether there is a greatest common factor (GCF) first.

Ex. 1 \(3x^4 - 33x^3 + 90x^2\)

- In this example the GCF is \(3x^2\).
- So when we factor, we have \(3x^2(x^2 - 11x + 30)\).
- Now we need to look at the polynomial remaining in the parentheses. Can this trinomial be factored into two binomials? In order to determine this make a list of all of the factors of 30.

<table>
<thead>
<tr>
<th>1</th>
<th>30</th>
<th>-1</th>
<th>-30</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>15</td>
<td>-2</td>
<td>-15</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>-3</td>
<td>-10</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>-5</td>
<td>-6</td>
</tr>
</tbody>
</table>

Since \(-5 + -6 = -11\) and \((-5)(-6) = 30\) we should choose \(-5\) and \(-6\) in order to factor the expression.

- The expression factors into \(3x^2(x - 5)(x - 6)\)

Note: Not all expressions will have a GCF. If a trinomial expression does not have a GCF, proceed by trying to factor the trinomial into two binomials.

II. Applying the difference of squares: \(a^2 - b^2 = (a - b)(a + b)\)

Ex. 2 \(4x^3 - 100x\)  
\[
4x(x^2 - 25) \\
4x(x - 5)(x + 5)
\]

Since \(x^2\) and 25 are perfect squares separated by a subtraction sign, you can apply the difference of two squares formula.
PRACTICE SET 6

Factor each expression.

1. $3x^2 + 6x$
2. $4a^2b^2 - 16ab^3 + 8ab^2c$

3. $x^2 - 25$
4. $n^2 + 8n + 15$

5. $g^2 - 9g + 20$
6. $d^2 + 3d - 28$

7. $z^2 - 7z - 30$
8. $m^2 + 18m + 81$

9. $4y^3 - 36y$
10. $5k^2 + 30k - 135$
F. Radicals

To simplify a radical, we need to find the greatest perfect square factor of the number under the radical sign (the radicand) and then take the square root of that number.

\[ \sqrt{72} \]
\[ \sqrt{36} \cdot \sqrt{2} \]
\[ 6\sqrt{2} \]

\[ 4\sqrt{90} \]
\[ 4 \cdot \sqrt{9} \cdot \sqrt{10} \]
\[ 4 \cdot 3 \cdot \sqrt{10} \]
\[ 12\sqrt{10} \]

\[ \sqrt{48} \]
\[ \sqrt{16} \sqrt{3} \]
\[ 4\sqrt{3} \]

\[ 4\sqrt{12} \]
\[ 2\sqrt{12} \]
\[ 2\sqrt{4} \sqrt{3} \]
\[ 2 \cdot 2 \cdot \sqrt{3} \]
\[ 4\sqrt{3} \]

Ex. 3: \[ \sqrt{48} \]

This is not simplified completely because 12 is divisible by 4 (another perfect square)

PRACTICE SET 7

Simplify each radical.

1. \[ \sqrt{121} \]
2. \[ \sqrt{90} \]
3. \[ \sqrt{175} \]
4. \[ \sqrt{288} \]
5. \[ \sqrt{486} \]
6. \[ 2\sqrt{16} \]
7. \[ 6\sqrt{500} \]
8. \[ 3\sqrt{147} \]
9. \[ 8\sqrt{475} \]
10. \[ \sqrt{\frac{125}{9}} \]
G. Graphing Lines

I. Finding the Slope of the Line that Contains each Pair of Points.

Given two points with coordinates \((x_1, y_1)\) and \((x_2, y_2)\), the formula for the slope, \(m\), of the line containing the points is

\[
m = \frac{y_2 - y_1}{x_2 - x_1}.
\]

Ex. (2, 5) and (4, 1)

\[
m = \frac{1 - 5}{4 - 2} = \frac{-4}{2} = -2
\]

The slope is -2.

Ex. (-3, 2) and (2, 3)

\[
m = \frac{3 - 2}{2 - (-3)} = \frac{1}{5}
\]

The slope is \(\frac{1}{5}\)

PRACTICE SET 8

1. (-1, 4) and (1, -2)  
2. (3, 5) and (-3, 1)  
3. (1, -3) and (-1, -2)

4. (2, -4) and (6, -4)  
5. (2, 1) and (-2, -3)  
6. (5, -2) and (5, 7)
II. Using the Slope – Intercept Form of the Equation of a Line.
The slope-intercept form for the equation of a line with slope \( m \) and \( y \)-intercept \( b \) is \( y = mx + b \).

Ex. \( y = 3x - 1 \)

Slope: 3 \( y \)-intercept: -1

Place a point on the \( y \)-axis at -1.
Slope is 3 or 3/1, so travel up 3 on the \( y \)-axis and over 1 to the right.

Ex. \( y = -\frac{3}{4}x + 2 \)

Slope: \( -\frac{3}{4} \) \( y \)-intercept: 2

Place a point on the \( y \)-axis at 2.
Slope is -3/4 so travel down 3 on the \( y \)-axis and over 4 to the right. Or travel up 3 on the \( y \)-axis and over 4 to the left.

PRACTICE SET 9

1. \( y = 2x + 5 \)

Slope: _____ \( y \)-intercept: _____

2. \( y = \frac{1}{2}x - 3 \)

Slope: _____ \( y \)-intercept: _____
3. $y = \frac{-2}{5}x + 4$
   
   Slope: __________
   
   $y$-intercept: __________

4. $y = -3x$
   
   Slope: __________
   
   $y$-intercept __________

5. $y = -x + 2$
   
   Slope: __________
   
   $y$-intercept: __________

6. $y = x$
   
   Slope: __________
   
   $y$-intercept: __________
III. Using Standard Form to Graph a Line.

An equation in standard form can be graphed using several different methods. Two methods are explained below.

a. Re-write the equation in \( y = mx + b \) form, identify the \( y \)-intercept and slope, then graph as in Part II above.

b. Solve for the \( x \)- and \( y \)-intercepts. To find the \( x \)-intercept, let \( y = 0 \) and solve for \( x \). To find the \( y \)-intercept, let \( x = 0 \) and solve for \( y \). Then plot these points on the appropriate axes and connect them with a line.

**Ex.** \( 2x - 3y = 10 \)

a. Solve for \( y \).

\[
-3y = -2x + 10 \\
y = \frac{-2x + 10}{-3} \\
y = \frac{2}{3}x - \frac{10}{3}
\]

OR

b. Find the intercepts:

- Let \( y = 0 \):
  \[2x - 3(0) = 10\]
  \[2x = 10\]
  \[x = 5\]

- Let \( x = 0 \):
  \[2(0) - 3y = 10\]
  \[-3y = 10\]
  \[y = -\frac{10}{3}\]

So \( x \)-intercept is \((5, 0)\)  
So \( y \)-intercept is \((0, -\frac{10}{3})\)

On the \( x \)-axis place a point at 5.
On the \( y \)-axis place a point at \(-\frac{10}{3} = -3\frac{1}{3}\)
Connect the points with the line.
PRACTICE SET 10

1. $3x + y = 3$

2. $5x + 2y = 10$

3. $y = 4$

4. $4x - 3y = 9$
5. $-2x + 6y = 12$

6. $x = -3$
H. Regression and Use of the Graphing Calculator

Note: For guidance in using your calculator to graph a scatterplot and finding the equation of the linear regression (line of best fit), please see the calculator direction sheet included in the back of the review packet.

PRACTICE SET 11

1. The following table shows the math and science test scores for a group of ninth graders.

<table>
<thead>
<tr>
<th>Math Test Scores</th>
<th>60</th>
<th>40</th>
<th>80</th>
<th>40</th>
<th>65</th>
<th>55</th>
<th>100</th>
<th>90</th>
<th>85</th>
</tr>
</thead>
<tbody>
<tr>
<td>Science Test Scores</td>
<td>70</td>
<td>35</td>
<td>90</td>
<td>50</td>
<td>65</td>
<td>40</td>
<td>95</td>
<td>85</td>
<td>90</td>
</tr>
</tbody>
</table>

Let's find out if there is a relationship between a student's math test score and his or her science test score.

a. Fill in the table below. Remember, the variable quantities are the two variables you are comparing, the lower bound is the minimum, the upper bound is the maximum, and the interval is the scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Create the scatter plot of the data on your calculator.

c. Write the equation of the line of best fit.

d. Based on the line of best fit, if a student scored an 82 on his math test, what would you expect his science test score to be? Explain how you determined your answer. Use words, symbols, or both.

e. Based on the line of best fit, if a student scored a 53 on his science test, what would you expect his math test score to be? Explain how you determined your answer. Use words, symbols, or both.
2. Use the chart below of winning times for the women's 200-meter run in the Olympics below to answer the following questions.

<table>
<thead>
<tr>
<th>Year</th>
<th>Time (Seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>23.00</td>
</tr>
<tr>
<td>1968</td>
<td>22.50</td>
</tr>
<tr>
<td>1972</td>
<td>22.40</td>
</tr>
<tr>
<td>1976</td>
<td>22.37</td>
</tr>
<tr>
<td>1980</td>
<td>22.03</td>
</tr>
<tr>
<td>1984</td>
<td>21.81</td>
</tr>
<tr>
<td>1988</td>
<td>21.34</td>
</tr>
<tr>
<td>1992</td>
<td>21.81</td>
</tr>
</tbody>
</table>

a. Fill in the table below. Remember, the variable quantities are the two variables you are comparing, the lower bound is the minimum, the upper bound is the maximum, and the interval is the scale for each axis.

<table>
<thead>
<tr>
<th>Variable Quantity</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b. Create a scatter plot of the data on your calculator.

c. Write the equation of the regression line (line of best fit) below. Explain how you determined your equation.

d. The Summer Olympics will be held in London, England, in 2012. According to the line of best fit equation, what would be the winning time for the women's 200-meter run during the 2012 Olympics? Does this answer make sense? Why or why not?
TI-83 Plus/TI-84 Graphing Calculator Tips

How to ...

**...graph a function**
Press the $Y=$ key. Enter the function directly using the $X,T,\theta,n$ key to input $x$. Press the GRAPH key to view the function. Use the WINDOW key to change the dimensions and scale of the graph. Pressing TRACE lets you move the cursor along the function with the arrow keys to display exact coordinates.

**...find the $y$-value of any $x$-value**
Once you have graphed the function, press CALC 2nd TRACE and select 1:value. Enter the $x$-value. The corresponding $y$-value is displayed and the cursor moves to that point on the function.

**...find the maximum value of a function**
Once you have graphed the function, press CALC 2nd TRACE and select 4:maximun. You can set the left and right boundaries of the area to be examined and guess the maximum value either by entering values directly or by moving the cursor along the function and pressing ENTER. The $x$-value and $y$-value of the point with the maximum $y$-value are then displayed.

**...find the zero of a function**
Once you have graphed the function, press CALC 2nd TRACE and select 2:zero. You can set the left and right boundaries of the root to be examined and guess the value either by entering values directly or by moving the cursor along the function and pressing ENTER. The $x$-value displayed is the root.

**...find the intersection of two functions**
Once you have graphed the function, press CALC 2nd TRACE and select 5:intersect. Use the up and down arrows to move among functions and press ENTER to select two. Next, enter a guess for the point of intersection or move the cursor to an estimated point and press ENTER. The $x$-value and $y$-value of the intersection are then displayed.

**...enter lists of data**
Press the STAT key and select 1:Edit. Store ordered pairs by entering the $x$ coordinates in L1 and the $y$ coordinates in L2. You can calculate new lists. To create a list that is the sum of two previous lists, for example, move the cursor onto the L3 heading. Then enter the formula L1+L2 at the L3 prompt.
...plot data

Once you have entered your data into lists, press STAT PLOT 2nd Y= and select Plot1. Select On and choose the type of graph you want, e.g. scatterplot (points not connected) or connected dot for two variables, histogram for one variable. Press ZOOM and select 9:ZoomStat to resize the window to fit your data. Points on a connected dot graph or histogram are plotted in the listed order.

...graph a linear regression of data

Once you have graphed your data, press STAT and move right to select the CALC menu. Select 4:LinReg(ax+b). Type in the parameters L1, L2, Y1. To enter Y1, press VARS and move right to select the Y-VARS menu. Select 1:Function and then 1:Y1. Press ENTER to display the linear regression equation and Y= to display the function.

...draw the inverse of a function

Once you have graphed your function, press DRAW 2nd PRGM and select 8:DrawInv. Then enter Y1 if your function is in Y1, or just enter the function itself.

...create a matrix

From the home screen, press 2nd x^-1 to select MATRX and move right to select the EDIT menu. Select 1:[A] and enter the number of rows and the number of columns. Then fill in the matrix by entering a value in each element.

...solve a system of equations

Once you have entered the matrix containing the coefficients of the variables and the constant terms for a particular system, press \text{MATRX} \begin{bmatrix} 2 & 6 \\ 1 & 2 \\ \end{bmatrix}, move to MATH, and select B:rref. Then enter the name of the matrix and press ENTER. The solution to the system of equations is found in the last column of the matrix.

...generate lists of random integers

From the home screen, press MATH and move left to select the PRB menu. Select 5:RandInt and enter the lower integer bound, the upper integer bound, and the number of trials, separated by commas, in that order. Press \text{STO×} and L1 to store the generated numbers in List 1. Repeat substituting L2 to store a second set of integers in List 2.